Optimizing Resource Allocation with the Hungarian Method

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Abstract

This study explores the assignment problem, a critical optimization challenge in operations research, which focuses on the optimal allocation of tasks to agents to minimize costs or maximize efficiency. As businesses face increasingly complex operational demands, effective resource management becomes essential. The study begins with an overview of the characteristics and significance of the assignment problem, followed by a detailed mathematical formulation that includes decision variables, objective functions, and constraints. The Hungarian Method is presented as a systematic approach to solving the assignment problem. It is illustrated through practical examples demonstrating its applicability in real-world scenarios such as workforce allocation and logistics. The results confirm the method's effectiveness in achieving optimal assignments while minimizing costs. This study underscores the importance of mathematical modeling in operations research and its potential to enhance decision-making processes across various industries, ensuring efficient resource utilization and improved operational performance.

Keywords: Assignment problem, Optimization, Hungarian method, Resource management, Mathematical modeling

1. Introduction

The main objective of this study is to analyze the assignment problem within the context of operations research, focusing on its significance in optimizing resource allocation to minimize costs or maximize efficiency. Amid the rising operational complexities encountered by businesses, effective resource management becomes crucial. It aims to assign a set of tasks to a set of agents such that each task is performed by one agent, and each agent is assigned exactly one task. The objective function is typically to minimize the total cost or maximize efficiency, considering the constraints and the costs associated with each task-agent pairing [14,18]. This paper begins by outlining the characteristics of the assignment problem and its relevance in real-world scenarios, such as workforce allocation and logistics.

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As the operational business becomes more challenging and complex, it is important to control costs to generate the maximum profit. Hence, the assignment problem, as one of the important parts in operations research, could assist in portraying the scenario where the task or resources could be allocated to jobs or agents at most optimal [12,15].

To apply this method, we must first know the characteristics of the assignment problem. The first characteristic is a one-to-one assignment. According to [4], each worker is assigned to one task only whereby to ensure the resources are utilized efficiently. This characteristic is important, especially in scenarios that require optimal distribution [13]. Next are binary decision variables. This characteristic introduces simplicity and clarity into task-agent modelling. Binary variables (0 or 1) make the decision straightforward, either assigning a task (1) or not (0), avoiding any ambiguity from partial assignments. Through this approach, the assignment problem ensures that each task is completed by a single agent, enhancing efficiency and eliminating the risks associated with shared responsibilities [11]. Lastly is an integer programming formulation. Typically solved using binary integer programming (BIP), the assignment problem captures the need for optimization in a structured model comprising an objective function and constraints. This integer programming approach effectively manages decision variables, maximizing output within defined limits.

There are real-world cases which suitable for optimizing the resources by minimizing cost as well as maximizing profit [9]. For example, in the transportation aspect, the drivers' average has wasted an estimation of 107 hours a year because of heavy traffic on the road. Moreover, approximately $2243 needs to be paid per year due to not efficiently assigning a vehicle-appropriate parking place [19]. This case aligns with the assignment problem for employees, which emphasizes the optimum allocation of limited resources and balancing the preferences with demands or needs.

The second real-world case scenario is congestion costs in Cairo. A 2010 review funded by the World Bank assessed the economic cost of vehicle congestion in Cairo. With an annual expense of approximately 47 billion Egyptian Pounds (around 8 billion USD), traffic congestion in Greater Cairo incurred a per capita cost of about 2,400 LE (400 USD), consuming around 15% of the per capita GDP [1]. This finding highlights the significant economic impact of traffic congestion and underscores the need for optimized vehicle assignments to reduce travel times, fuel costs, and economic losses [21]. Another real-world case scenario is the impact of the Covid-19 pandemic on e-commerce logistics. The pandemic accelerated the shift to online shopping, heightening the demand for efficient logistics services. The assignment problem model helps companies like Anteraja streamline their operations, ensuring that each courier's route is optimized to meet delivery demands promptly and cost-effectively [16, 20].

The assignment problem is mathematically formulated using decision variables, objective functions, and constraints. The Hungarian method is introduced as an effective solution approach [3, 6]. By applying this method to various examples, including employee-task assignments, the paper demonstrates its practicality and effectiveness in minimizing costs. The justification for this study lies in its potential to enhance decision-making processes across industries by employing mathematical modelling for efficient resource utilization [3].

2. Method

The assignment problem is a fundamental optimization challenge commonly found in resource allocation scenarios. It involves assigning a set of tasks to a group of agents in such a way that either the total cost is minimized or the overall efficiency is maximized. This problem can be modeled mathematically using decision variables, an objective function, and a set of constraints. The key

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components of the assignment problem and its structure are presented below.

Decision Variables

In the assignment problem, the binary decision variables is used to represent whether a particular task is assigned to a particular agent. Specifically, let:

$$X_{ij} = \begin{cases} 1 \text{ if task } i \text{ is assigned to agent } j \\ 0, \text{ otherwise} \end{cases}$$

Here, X_{ij} takes the value of 1 if task *i* is assigned to agent *j* and 0 if it is not. This binary representation simplifies the problem by ensuring that each task-agent pair is either chosen or not, with no fractional assignments.

Objective Function

The objective of the assignment problem is either to minimize the total cost of assigning tasks to agents or to maximize the overall efficiency of these assignments.

a) Minimizing Cost

To minimize the total cost, we introduce a cost matrix C_{ij}, where C_{ij} represents the cost of assigning task i to agent j. The objective is to minimize the total cost for all assignments, which can be expressed as:

$$\text{Minimize } Z = \sum i \sum j \, C_{ij} + X_{ij}$$

This objective function sums up the product of the cost C_{ij} and the binary decision variable X_{ij} for all possible task-agent combinations. The goal is to minimize the overall sum of these products, thereby achieving the least total cost.

b) Maximizing Efficiency

In some cases, the goal may not be to minimize cost but rather to maximize efficiency. We introduce an efficiency matrix E_{ij}, where E_{ij} denotes the efficiency of assigning task i to agent j. In this case, the objective function becomes:

$$\text{Maximize } Z = \sum i \sum j \, E_{ij} + X_{ij}$$

Here, the function aims to maximize the sum of the efficiency values for all assignments, ensuring that tasks are allocated to agents in a way that optimizes performance.

Constraints

The assignment problem is subject to several constraints, ensuring that tasks and agents are allocated appropriately [7, 17].

a) Task Assignment Constraint: Each task must be assigned to exactly one agent. This ensures that no task is left unassigned or assigned to multiple agents. Mathematically, this is represented as:

$$\sum i \, X_{ij} = 1 \text{ for all } i$$

This constraint guarantees that for each task i, only one of the Xij variables will take the value 1, meaning that task i is assigned to exactly one agent.

b) Agent Assignment Constraint: Each agent must be assigned to exactly one task. This prevents any

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agent from being assigned to multiple tasks or remaining unassigned. This can be written as:

$$\sum j X_{ij} = 1 \text{ for all } j$$

This constraint ensures that for each agent j, only one task will be assigned.

c) Binary Constraint: The decision variables X_{ij} are binary, meaning they can only take values of 0 or 1. This is expressed as:

$$X_{ij} \in \{0,1\} \text{ for all } i,j$$

This condition ensures that the assignment is either made (1) or not made (0) for each task-agent pair.

d) Non-Negativity Condition

Although the decision variables X_{ij} are binary, ensuring that they are either 0 or 1, the mathematical formulation includes a non-negativity condition to emphasize that the values cannot be negative. This condition is essential for some optimization algorithms.

In mathematical terms, the model can be expressed as follows:

Objective Function (for minimizing cost):

$$\text{Minimize } Z = \sum i \sum j C_{ij} + X_{ij} \text{ or (for maximizing efficiency): Maximize } Z = \sum i \sum j E_{ij} + X_{ij}$$

Constraints:

$$\sum i X_{ij} = 1 \quad \text{for all } i$$

$$\sum j X_{ij} = 1 \quad \text{for all } j$$

$$X_{ij} \in \{0,1\} \text{ for all } i,j$$

This mathematical formulation provides a clear, structured approach to solving assignment problems in various real-world applications, from workforce allocation to resource distribution. The assignment problem is a classic optimization problem that can be effectively solved using mathematical modeling. By defining decision variables, formulating an appropriate objective function (either to minimize cost or maximize efficiency), and applying necessary constraints, the problem can be solved to achieve optimal task-agent assignments. This model is widely applicable across industries, including logistics, project management, and human resources, ensuring efficient and cost-effective operations.

3. Results and Discussion

In today's fast-paced business environment, efficient task allocation is crucial for optimizing resources and minimizing costs. One such method for solving assignment problems is the Hungarian method, a mathematical approach that ensures optimal assignment of tasks while minimizing costs [2, 8, 22]. This section presents a step-by-step solution to a classic assignment problem using the Hungarian method. The scenario involves assigning four employees to four tasks to minimize the total cost of the assignment.

3.1 Example 1

The initial cost matrix illustrates the cost associated with assigning each task to a specific employee. The goal is to minimize the total assignment cost while ensuring that each employee is assigned to exactly one task. Table 1 serves as the starting point for applying the Hungarian method.

Table 1. Initial Problem Matrix for Employee-Task Assignments

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Employee/ Task	1	2	3	4
A	5	3	2	4
B	4	2	3	5
C	3	4	5	2
D	2	5	4	3

This matrix will undergo systematic transformations, which are row reduction, column reduction, and adjustments to simplify the problem and identify the optimal assignment. Each step reduces the overall complexity, ensuring a cost-effective solution.

Step 1: Row Reduction

The first step in the Hungarian Method involves row reduction. This process entails subtracting the smallest element in each row from every element in that row. This adjustment helps simplify the problem by ensuring that each row contains at least one zero, which facilitates optimal assignment later.

a. For Employee A (Row 1), the smallest value is 2. Subtracting 2 from all elements in the row gives us: 5-2, 3-2, 2-2, 4-2 = 3, 1, 0, 2
b. For Employee B (Row 2), the smallest value is also 2. Subtracting 2 from all elements results in: 4-2, 2-2, 3-2, 5-2 = 2, 0, 1, 3
c. For Employee C (Row 3), the smallest value is 2 as well. Subtracting 2 yields: 3-2, 4-2, 5-2, 2-2 = 1, 2, 3, 0
d. For Employee D (Row 4), the smallest value is 2. Subtracting this from each element provides: 2-2, 5-2, 4-2, 3-2 = 0, 3, 2, 1

Row reduction is the first step in the Hungarian method. The smallest element in each row is subtracted from every element in that row, ensuring each row has at least one zero. Table 2 below shows the updated matrix after this step.

Table 2. Cost Matrix After Row Reduction

Employee/ Task	1	2	3	4
A	3	1	0	2
B	2	0	1	3
C	1	2	3	0
D	0	3	2	1

The resulting zeros simplify the problem by providing potential optimal assignments in each row. The matrix will now undergo column reduction for further simplification.

Step 2: Column Reduction

After reducing the rows, the next step is column reduction. This process involves subtracting the smallest element in each column from every element in that column. Similar to row reduction, this further simplifies the problem by introducing more zeros in the matrix.

a. In Column 1, the smallest value is 0, so no changes are needed.
b. In Column 2, the smallest value is 0, so no changes are required.
c. In Column 3, the smallest value is 0, and hence, no changes are necessary.

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d. In Column 4, the smallest value is 0, so the column remains unchanged.

Column reduction is performed next to ensure each column contains at least one zero. This step subtracts the smallest element in each column from all other elements in that column.

Table 3. Cost Matrix After Column Reduction

Employee / Task	**1**	**2**	**3**	**4**
A	3	1	0	2
B	2	0	1	3
C	1	2	3	0
D	0	3	2	1

The matrix now contains multiple zeros across rows and columns, enabling the identification of optimal assignments using the covering zeros approach.

Step 3: Optimal Assignment (Covering Zeros)

The next stage in the Hungarian method is to make the optimal assignment by covering all zeros in the matrix with the minimum number of horizontal and vertical lines. Each zero represents a potential assignment of a task to an employee, and we aim to assign one task to each employee while ensuring minimal cost.

The following zeros in the matrix are observed:

a. Employee A has a zero in column 3.
b. Employee B has a zero in column 2.
c. Employee C has a zero in column 4.
d. Employee D has a zero in column 1.

Using these observations, the following assignments are made:

a. Assign Employee A to Task 3.
b. Assign Employee B to Task 2.
c. Assign Employee C to Task 4.
d. Assign Employee D to Task 1.

This assignment covers all zeros in the matrix, confirming that it is an optimal solution.

Step 4: Calculating the Total Cost

The final step is to calculate the total cost based on the original cost matrix. The optimal assignments and their corresponding costs are as follows.

a. Employee A assigned to Task 3: Cost = 2
b. Employee B assigned to Task 2: Cost = 2
c. Employee C assigned to Task 4: Cost = 2
d. Employee D assigned to Task 1: Cost = 2

Thus, the total cost of this optimal assignment is: 2 + 2 + 2 + 2 = $8. By following the Hungarian method, the employee-task assignment problem is solved, ensuring that each employee is assigned exactly one task, and each task is assigned to exactly one employee. The method allowed us to minimize the total cost of assignments to 8, demonstrating the efficiency of the Hungarian Method in solving such optimization problems. To validate the manual solution derived from the Hungarian

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method, the same problem was solved using Excel Solver. Figure 1 presents the optimal assignment visualized through Excel's output.

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DATA TABLE

Employee / Task	1	2	3	4
A	5	3	2	4
B	4	2	3	5
C	3	4	5	2
D	2	5	4	3

RESULT TABLE

Employee / Task	1	2	3	4	Total		Supply
A	0	0	1	0	1	=	1
B	0	1	0	0	1	=	1
C	0	0	0	1	1	=	1
D	1	0	0	0	1	=	1
Requirement	1	1	1	1			
	=	=	=	=			TOTAL
Limit	1	1	1	1			8

Figure 1. Optimal Task Assignments Visualized Using Excel Solver

The Excel Solver result confirms the manual solution, with each task optimally assigned to an employee at the minimum total cost. This demonstrates the reliability and efficiency of both manual and software-based approaches. Therefore, it is proven that the result is optimal as below:

Employee A → Task 3
Employee B → Task 2
Employee C → Task 4
Employee D → Task 1
Costs: $8

3.2 Example 2

The balanced assignment problem introduces additional tasks or agents to ensure the matrix is square. Table 4 below shows the initial matrix for this scenario, which will be solved using the Hungarian method.

Table 4. Initial Matrix for Balanced Assignment Problem

Machine / Job	1	2	3	4	5	Minimum
1	13	8	16	18	19	8
2	9	15	24	9	12	9
3	12	9	4	4	4	4
4	6	12	10	8	13	6
5	15	17	18	12	20	12

This matrix will undergo a series of transformations, similar to the earlier example, to achieve an optimal assignment that balances costs and resources.

1. The first step in solving the balanced assignment problem is row subtraction, where the smallest value in each row is subtracted from all other elements in the same row, as per Table 5.

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Table 5. Cost Matrix After Row Subtraction in Balanced Assignment Problem

Job \ Machine	1	2	3	4	5
1	(13-8) 5	(8-8) 0	(16-8) 8	(18-8) 10	(19-8) 11
2	(9-9) 0	(15-9) 6	(24-9) 15	(9-9) 0	(12-9) 3
3	(12-4) 8	(9-4) 5	(4-4) 0	(4-4) 0	(4-4) 0
4	(6-6) 0	(12-6) 6	(10-6) 4	(8-6) 2	(13-6) 7
5	(15-12) 3	(17-12) 5	(18-12) 6	(12-12) 0	(20-12) 8
Minimum	0	0	0	0	0

The resulting matrix simplifies the problem and prepares it for the subsequent column reduction step.

2. Following row subtraction, column subtraction is performed to ensure that at least one zero exists in each column. Table 6 below shows the updated matrix after this step.

Table 6. Cost Matrix After Column Subtraction in Balanced Assignment Problem

Job \ Machine	1	2	3	4	5
1	(5-0) 5	(0-0) 0	(8-0) 8	(10-0) 10	(11-0) 11
2	(0-0) 0	(6-0) 6	(15-0) 15	(0-0) 0	(3-0) 3
3	(8-0) 8	(5-0) 5	(0-0) 0	(0-0) 0	(0-0) 0
4	(0-0) 0	(6-0) 6	(4-0) 4	(2-0) 2	(7-0) 7
5	(3-0) 3	(5-0) 5	(6-0) 6	(0-0) 0	(8-0) 8

With zeros now present in each row and column, the matrix is ready for the line-covering step to identify optimal assignments.

3. Table 7 illustrates the result after applying the line-covering method to the cost matrix. Minimum lines are drawn to cover all zeros in the matrix, facilitating the identification of assignments.

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Table 7. Matrix with Minimum Lines to Cover All Zeros

Job \ Machine	1	2	3	4	5
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

The number of lines drawn is equal to the order of the matrix, confirming that an optimal solution can now be derived by adjusting uncovered elements.

4. Determine the smallest uncovered element x. The smallest uncovered element, x = 3. The smallest uncovered value in the matrix is identified and used to adjust the uncovered and covered elements. Table 8 below presents the updated matrix after these adjustments.

Table 8: Matrix Showing Uncovered Minimum Value and Adjustments.

Job \ Machine	1	2	3	4	5
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

The smallest uncovered, x

This adjustment step prepares the matrix for further iterations, ultimately leading to an optimal assignment solution.

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5. Then, repeat step 3, subtract the minimum value from all values in the uncovered element, and draw a minimum number of horizontal and vertical values to cover all zeros.

After several iterations of adjustments and line-covering, the matrix is reduced to its final form. Table 9 below shows the optimized cost matrix.

Table 9. Final Cost Matrix After Adjustments in Balanced Assignment Problem

Job \ Machine	1	2	3	4	5
1	5	0	5	10	8
2	0	6	12	0	0
3	8	5	0	0	0
4	0	6	1	2	4
5	3	5	3	0	5

The zeros in this matrix represent the optimal assignments for each task-agent pair, minimizing total cost.

6. Table 9 shows that no of lines drawn to cover all zeroes = 5 and order of matrix = 5. Therefore, we can form an assignment. Table 10 summarizes the optimal assignments of machines to jobs based on the final cost matrix, along with the corresponding costs for each assignment.

Table 10. Optimized Machine-Job Assignments and Corresponding Costs

Job	Machine	Cost
1	2	8
2	5	12
3	3	4
4	1	6
5	4	12

The total cost of the assignment demonstrates the effectiveness of the Hungarian method in solving balanced assignment problems. To recapitulate, total cost = 8+12+4+6+12 = $42. Figure 2 below represents the network of machine-job assignments, highlighting the connections and corresponding.

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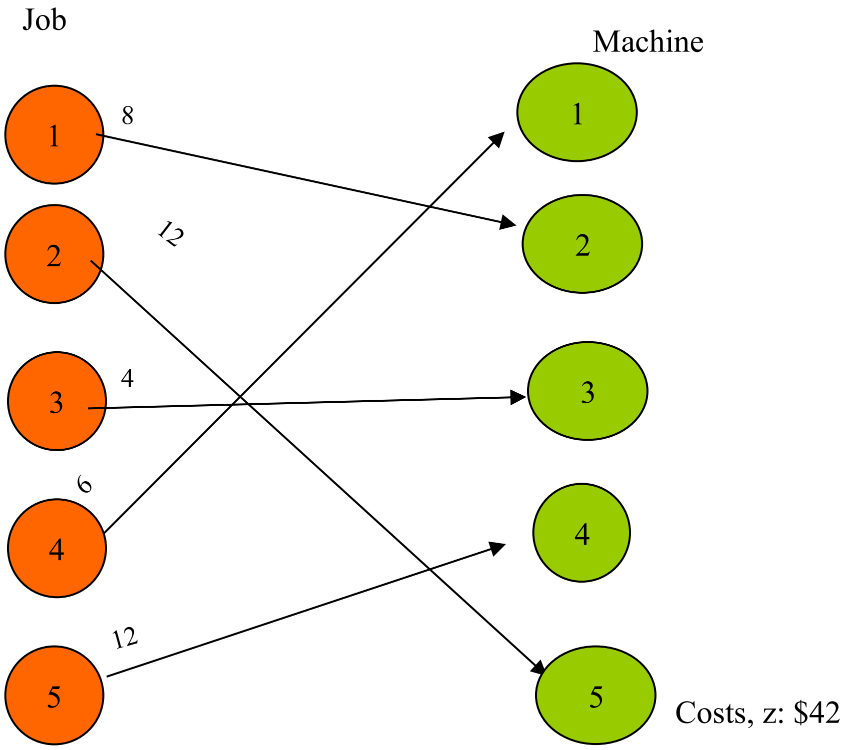

Figure 2. Network Diagram for Balanced Assignment Problem

This diagram provides a clear visualization of the optimized assignments, reinforcing the results derived from the cost matrix. We use a software tool (e.g., Excel Solver) to verify the optimal solution. Figures 3 and 4 are the result and report by Excel Solver:

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DATA TABLE

Job/ Machine	1	2	3	4	5
A	13	8	16	18	19
B	9	15	24	9	12
C	12	9	4	4	4
D	6	12	10	8	13
E	15	17	18	12	20

RESULT TABLE

Job / Machine	1	2	3	4	5	Total		Supply
A	0	1	0	0	0	1	=	1
B	0	0	0	0	1	1	=	1
C	0	0	1	0	0	1	=	1
D	1	0	0	0	0	1	=	1
E	0	0	0	1	0	1	=	1
Requirement	1	1	1	1	1			
	=	=	=	=	=			TOTAL
Limit	1	1	1	1	1			42

Figure 3. **Solution for Balanced Assignment Problem Using Excel Solver**

Solver Engine
Engine: Simplex LP
Solution Time: 0.031 Seconds.
Iterations: 17 Subproblems: 0
Solver Options
Max Time 100 sec, Iterations 100, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 5%, S

Objective Cell (Min)

Cell	Name	Original Value	Final Value
I22	Limit TOTAL	63	42

Variable Cells

Cell	Name	Original Value	Final Value	Integer
B15	A	0	0	Integer
C15	A	0	1	Integer
D15	A	1	0	Integer
E15	A	0	0	Integer
F15	A	0	0	Integer
B16	B	0	0	Integer
C16	B	1	0	Integer
D16	B	0	0	Integer
E16	B	0	0	Integer
F16	B	0	1	Integer
B17	C	1	0	Integer
C17	C	0	0	Integer
D17	C	0	1	Integer
E17	C	0	0	Integer
F17	C	0	0	Integer
B18	D	0	1	Integer
C18	D	0	0	Integer
D18	D	0	0	Integer
E18	D	0	0	Integer
F18	D	0	0	Integer
B19	E	0	0	Integer
C19	E	0	0	Integer
D19	E	0	0	Integer
E19	E	0	1	Integer
F19	E	1	0	Integer

Figure 4. **Results Report from Excel Solver for Balanced Assignment Problem**

The solver results align with the manual calculations, verifying the accuracy of the Hungarian method for solving balanced assignment problems. This report provides quantitative confirmation of the solution, offering an additional layer of validation for the optimization process. From this result, we conclude that the optimal solution is z = $42, and the job-machine assignments are identical between manual calculations and the Excel Solver output. This consistency confirms both the reliability of the Hungarian method and the precision of the Solver tool in addressing balanced assignment problems, ensuring the process is accurate and efficient for practical applications.

3.3 Example 3

The student internship director needs as much coverage as possible in the office next week. The more hours that can be put in each day, the better. She has asked the students to provide a list of how

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many hours they are available each day of the week. Each student can be there on only one day, and there must be a student in the office each day of the week. Use Table 11 provided to determine the schedule that gives the most coverage (BrainKart, 2018).

Table 11. Problem Matrix for Student Schedule Assignment

Student	Mon	Tue	Wed	Thurs	Fri
A	2	4	8	4	6
B	3	2	7	3	2
C	6	8	6	5	4
D	7	4	3	6	8
E	4	5	3	1	4

This initial matrix will undergo a series of transformations, similar to the task assignment problem, to determine the optimal schedule. To do the assignment problem using the Hungarian method, we must follow these steps:

i) The first step in solving the student scheduling problem is row reduction as per Table 12. In this step, the smallest value in each row is subtracted from all elements in that row. This operation ensures that at least one zero exists in each row, simplifying the assignment process.

Table 12. Row-Reduced Cost Matrix for Student Scheduling Problem

Student	Mon	Tue	Wed	Thurs	Fri
A	2	4	8	4	6
B	3	2	7	3	2
C	6	8	6	5	4
D	7	4	3	6	8
E	4	5	3	1	4

The resulting row-reduced matrix provides a basis for further transformations. This step sets the stage for column reduction, the next step in the Hungarian method.

ii) The next step in the Hungarian method is column reduction. Here, the smallest value in each column is subtracted from all elements in that column. The resulting matrix ensures that at least one zero exists in each column as per Table 13 below.

Table 13. Column-Reduced Cost Matrix for Student Scheduling Problem

Student	Mon	Tue	Wed	Thurs	Fri
A	**0**	2	6	2	4
B	1	**0**	5	1	**0**
C	2	4	2	1	0
D	4	1	**0**	3	5

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E	3	4	2	**0**	3

Since the lowest number of each column is 0, the result is the same. The column-reduced matrix now contains zeros that represent potential optimal assignments. This matrix will be used to cover zeros and identify feasible assignments.

iii) Find 0 from each row, and the remaining 0 of the same row/column need to be canceled out. Table 14 below shows the result after identifying and covering zeros. Rows and columns containing zeros are canceled out to avoid conflicts in assignments, ensuring that each student is assigned to only one day.

Table 14. Matrix with Canceled Zeros for Student Scheduling Problem

Student	Mon	Tue	Wed	Thurs	Fri
A	**0**	2	6	2	4
B	1	**0**	5	1	--0--
C	2	4	2	1	**0**
D	4	1	**0**	3	5
E	3	4	2	**0**	3

Since we managed to get 0 for each student and assigned to the selected day, we can conclude that this method. This step demonstrates how zeros are systematically managed to derive optimal assignments. The final adjustments will refine the matrix further for complete optimization.

iv) The following Table 15 presents the final optimal assignment of students to days after completing all steps of the Hungarian method. This result ensures maximum coverage while adhering to the constraints of the problem. Therefore, we can say that the days have been assigned to the student as below:

Table 15. Final Optimal Assignment of Students to Days

Mon	Tue	Wed	Thurs	Fri
Student A	Student B	Student D	Student E	Student C

The optimized schedule shows how each student is assigned to a specific day, maximizing coverage and minimizing conflicts. The result highlights the efficiency of the Hungarian method in solving real-world scheduling problems.

v) To ensure accuracy, the student scheduling problem was solved using Excel Solver. Figure 5 below illustrates the optimal assignment of students to days:

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DATA TABLE

Student	Mon	Tue	Wed	Thurs	Fri
A	2	4	8	4	6
B	3	2	7	3	2
C	6	8	6	5	4
D	7	4	3	6	8
E	4	5	3	1	4

RESULT TABLE

Student	Mon	Tue	Wed	Thurs	Fri	Total		Supply
A	1	0	0	0	0	1	=	1
B	0	1	0	0	0	1	=	1
C	0	0	0	0	1	1	=	1
D	0	0	1	0	0	1		1
E	0	0	0	1	0	1	=	1
Requirement	1	1	1	1	1			
	=	=	=	=	=			TOTAL
Limit	1	1	1	1	1			12

Figure 5. Optimal Student Scheduling Solution Generated by Excel Solver

The solver's output confirms the manual solution, showcasing the utility of Excel Solver in optimizing scheduling problems. So, it is proven that the result is optimal. Both methods effectively solve the assignment problem with the same optimal cost.

Student A → Monday
Student B → Tuesday
Student C → Friday
Student D → Wednesday
Student E → Thursday
Costs: $12

5. Conclusion

The research findings highlight the pivotal role of mathematical optimization in operations research, demonstrating its capacity to address complex assignment problems with precision and efficiency. By offering a structured framework for decision-making, this study significantly enhances operational performance and cost-effectiveness across diverse sectors. Its implications extend beyond theoretical advancements, showcasing real-world applications in logistics, project management, and human resource management. It emphasizes the transformative impact of optimal task-agent assignments, underscoring their importance in achieving organizational goals, maximizing resource utilization, and driving strategic success in an increasingly competitive environment.

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Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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